

# Some Remarks on the Search for CP Violation in Z-Decays <sup>1</sup>

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## Abstract:

We discuss some issues arising in the search for CP violation in the decays  $Z \rightarrow \tau^+\tau^-$  and  $Z \rightarrow b\bar{b}X$ . The form-factor and the effective Lagrangian approach to parametrize CP-violating effects are compared. We emphasize the interest to study both real and imaginary parts of CP-violating form factors like the weak dipole moment form factor  $d_\tau^Z(m_Z^2)$  of the  $\tau$ -lepton. We propose an “optimal” way to search for CP violation when  $Z \rightarrow b\bar{b}X$  events have to be selected on a statistical basis from the total number of  $Z$  hadronic decays.

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# 1 Introduction

The anomalous magnetic dipole moments (AMM) of the electron and muon have played an important role in precision tests of quantum electrodynamics. Future measurements of the muon AMM may eventually become sensitive to quantum corrections as predicted by the Standard Model (SM) [1]. Searches for non-zero electric dipole moments (EDM), for instance of the electron or the neutron, are motivated by the sensitivity of these moments to CP-violating interactions beyond the one parametrized by the phase in the Kobayashi-Maskawa matrix of the SM. In precise terms these dipole moments are the chirality-flipping form factors in the fermion photon vertex at zero momentum transfer. The fermion Z boson vertex contains analogous chirality-flipping form factors (see below). In renormalizable theories these moments cannot appear as elementary couplings but are induced by quantum corrections. It can be argued that the tau dipole form factors are probably much more sensitive to new physics than the corresponding moments of the electron or muon. For instance, diagrams with exchange of a neutral Higgs boson in the gauge boson fermion vertex induce contributions to the dipole form factors which scale with the third power of the external fermion mass. It is therefore of great interest to determine the tau moments as precisely as possible [2]-[8].

Most direct or indirect experimental determinations which were made so far for the tau moments involve time like momentum transfers. In particular the most precise direct determination to date is that of the weak dipole moment (WDM) form factor of the tau at  $q^2 = m_Z^2$  in ref. [9] which combines the results of the OPAL [10], [11] and ALEPH collaborations [12], [13]. The DELPHI collaboration [14] obtained upper limits on the AMM and EDM form factors of the tau lepton at  $q^2 = 0$  from an analysis of  $e^+e^- \rightarrow \tau^+\tau^-\gamma$  events.

For  $q^2 = m_Z^2$  the form factors can be complex. On the other hand, effects of new physics beyond the SM, in particular of new CP-violating interactions with an intrinsic energy scale  $\Lambda \gg m_Z$  can quite generally be parametrized by an effective Lagrangian  $\mathcal{L}_{\text{eff}}$ . This Lagrangian can be expanded in terms of hermitian operators of dimensions 5,6,... constructed from SM fields. The coupling constants multiplying the hermitian operators in  $\mathcal{L}_{\text{eff}}$  must be real. How do these real coupling constants relate to the complex form factors above?

In this note we recall for the convenience of the experimentalists the relation between the coupling constants of the effective Lagrangian and the form factors. We discuss the definitions and a few properties of these form factors. Most experiments used the observable  $T_{33}$  [3, 5] which determines the real part of the weak dipole moment form factor. But we emphasize that experiments which are sensitive to form factors at time like momentum transfers should measure both their real and imaginary parts, as was done for the WDM form factor of the tau lepton in [11]. Furthermore, we make some remarks concerning an “optimal” strategy for searching for CP violation when one has to select a class of events on a statistical basis. As an example

we discuss the CP violation search in  $Z \rightarrow b\bar{b}X$  (cf. [15]).

## 2 Form Factors

Precision measurements of the electromagnetic moments of the electron and muon amount to measuring the form factors of the respective electromagnetic current matrix element at zero momentum transfer. Using Lorentz covariance and current conservation, the matrix element of the electromagnetic current for a lepton  $f = e, \mu, \tau$  can be written as follows (for a discussion of form factors and CP violation cf. e.g. [16]):

$$\begin{aligned} \langle f(p') | e J_\mu^{em}(0) | f(p) \rangle &= -e \bar{u}(p') \Gamma_\mu u(p), \\ (f = e, \mu, \tau). \end{aligned} \quad (1)$$

Here  $e J_\mu^{em}$  is the electromagnetic current operator,  $e$  is the positron charge ( $e > 0$ ) and the vertex factor  $\Gamma_\mu$  can be decomposed as:

$$\Gamma_\mu = F_1 \gamma_\mu + F_2 i \sigma_{\mu\nu} q^\nu / 2m_f + F_3 \sigma_{\mu\nu} \gamma_5 q^\nu / 2m_f + F_4 (\gamma_\mu \gamma_5 q^2 - 2m_f \gamma_5 q_\mu), \quad (2)$$

where  $m_f$  is the mass of the fermion  $f$ ,  $q = p' - p$ , and  $F_i = F_i(q^2)$ . Our metric and  $\gamma$ -matrix conventions are as in [17]. The static quantities  $F_i(0)$  ( $i = 1, 2, 3$ ) which are measured in the soft photon limit  $q \rightarrow 0$  are real and  $SU(2) \times U(1)$  gauge-invariant. They are definable as the residues of the photon pole  $\propto 1/q^2$  in physical scattering amplitudes. Recall that by charge renormalization  $F_1(0) = 1$ . The magnetic moment  $\mu_f$ , the AMM  $a_f^\gamma$ , and the P- and T-violating EDM  $d_f^\gamma$  are given by

$$\begin{aligned} \mu_f &= -\frac{e}{2m_f} [F_1(0) + F_2(0)], \\ a_f^\gamma &= F_2(0), \\ d_f^\gamma &= \frac{e}{2m_f} F_3(0). \end{aligned} \quad (3)$$

The form factor  $F_4(0)$  is related to the ‘‘anapole moment’’ of the fermion  $f$  [18]. The anapole moment operator  $\vec{\mathcal{A}}$  is defined by

$$\vec{\mathcal{A}}(t) = -e\pi \int d^3x |\vec{x}|^2 \vec{J}^{em}(\vec{x}, t), \quad (4)$$

and the anapole moment  $\mathcal{A}_f$  of the fermion  $f$  by

$$\langle f(p', s') | \vec{\mathcal{A}}(0) | f(p, s) \rangle \Big|_{p'=p=p_R} = \mathcal{A}_f \langle f(p', s') | \vec{S} | f(p, s) \rangle \Big|_{p'=p=p_R} \quad (5)$$

Here  $\vec{S} = \vec{\sigma}/2$  denotes the spin operator and  $p_R$  the 4-momentum vector of the fermion  $f$  at rest. A simple calculation gives

$$\mathcal{A}_f = 8\pi e F_4(0). \quad (6)$$

The anapole moment is a P-odd and T-even quantity. When defined as in (6), it is invariant under electromagnetic gauge transformations but not under the full  $SU(2) \times U(1)$  gauge transformations of the SM. (Its SM value was computed in [19] in the general  $R_\xi$  gauge.) This can be seen as follows. The form factor  $F_4$  in (2) multiplies a vector structure  $\gamma_\mu \gamma_5 q^2 - 2m_f q_\mu \gamma_5$  which does not lead to a pole term in  $q^2$  from photon exchange in physical amplitudes. Indeed, multiplying with the photon propagator  $\propto 1/q^2$  we get

$$\gamma^\mu \gamma_5 - 2m_f \frac{q^\mu}{q^2} \gamma_5.$$

The first term has no pole and the term  $q^\mu/q^2$  gives a vanishing contribution due to current conservation when contracted with the vertex on the other side of the propagator line (Fig. 1). Thus the anapole moment corresponds to a contact interaction at small  $|q^2|$  and its value is only fixed relative to the convention on  $\gamma - Z$  mixing in their  $2 \times 2$  propagator matrix.

In [20] the anapole moment is defined as being the axial vector contact interaction with an external electromagnetic current. This definition includes corrections from both photon and Z boson exchange and leads to an expression which is gauge-invariant with respect to the full electroweak theory.

For  $q^2 \neq 0$  the form factors  $F_i(q^2)$  have infrared divergences due to ordinary QED radiative corrections. We will ignore these infrared divergences for the moment.

Tau pair production in  $e^+e^-$  collisions is described by the corresponding S matrix element. It involves in particular the vertex  $\gamma^*(q) \rightarrow \bar{f}(p) + f(p')$  where now  $q = p' + p$  and the vertex where the  $\gamma^*$  is replaced by a Z. The  $\gamma^*$  vertex is obtained from (1),(2) by crossing. The form factors  $F_i(q^2)$  develop imaginary parts for  $q^2 > s_0$ . The threshold value  $s_0$  may depend on the form factor. Usually one has normal thresholds in which case  $s_0 = (m_1 + m_2)^2$  where  $m_1, m_2$  are the masses of the lightest pair of intermediate particles, which can couple to  $f\bar{f}$  and the current in question.  $\text{Re}F_i$  and  $\text{Im}F_i$  are related by a dispersion integral. For  $q^2 \neq 0$  the form factors are in general not invariant with respect to the gauge fixing conventions of the electroweak theory.

Gauge-invariant expressions at the Z resonance are obtained from a Laurent expansion of the S matrix element at its Z pole in the complex  $s$  plane. For the discussion of CP violation effects at the Z resonance it is legitimate to neglect the non-resonant terms and to study the residue of the pole in a weak coupling expansion, typically to one-loop order. Then the S matrix element for the on-shell process  $Z \rightarrow \tau^+ \tau^-$  applies. It can be decomposed as follows:

$$\langle \tau^-(p_1) \tau^+(p_2) | S | Z(p, \epsilon) \rangle = i(2\pi)^4 \delta(p - p_1 - p_2) e \epsilon^\mu \bar{u}(p_1) \Lambda_\mu v(p_2), \quad (7)$$

where

$$\Lambda_\mu = G_1(m_Z^2) \gamma_\mu + G_2(m_Z^2) i \sigma_{\mu\nu} p^\nu / 2m_\tau + G_3(m_Z^2) \sigma_{\mu\nu} \gamma_5 p^\nu / 2m_\tau + G_4(m_Z^2) \gamma_\mu \gamma_5, \quad (8)$$

and  $p = p_1 + p_2$ . Note the following:

- 1) The vertex function (8) is electroweak gauge-invariant if all the particles are on-shell.
- 2) The vertex function for the coupling of  $f\bar{f}$  to an off-shell  $Z$  boson with invariant mass squared  $p^2 = s$  is again given by (8) but with  $G_i(m_Z^2) \rightarrow G_i(s)$ . Also the term  $iG_5(s)p_\mu + G_6(s)p_\mu\gamma_5$  containing two more form factors has to be added in (8). However, the form factors  $G_{5,6}$  do not contribute to  $e^+e^- \rightarrow \tau^+\tau^-$  in the limit of vanishing electron mass.
- 3) The form factors  $G_i(p^2 = m_Z^2)$  are in general complex. The form factors  $G_3$  and  $G_5$  are CP-violating.
- 4) For a check of the time reversal symmetry T the decay reaction  $Z \rightarrow f\bar{f}$  has to be compared with the production reaction  $f\bar{f} \rightarrow Z$ . The form factors for the latter reaction can be obtained from (8) assuming CPT invariance or crossing symmetry, which are, of course, both valid in a local relativistic quantum field theory (cf. e.g. [21]). Thus, the reaction  $Z \rightarrow f\bar{f}$  by itself allows straightforward CP-tests, whereas for T-tests one has to assume CPT invariance and then T invariance is equivalent to CP invariance.
- 5) In analogy to the electromagnetic moments at  $q^2 = 0$  we can define electromagnetic and weak anomalous magnetic moment (AMM, WMM) as well as electric and weak dipole moment (EDM, WDM) form factors by

$$\begin{aligned} a_\tau^\gamma(s) &= F_2(s), \\ a_\tau^Z(s) &= G_2(s), \end{aligned} \tag{9}$$

$$\begin{aligned} d_\tau^\gamma(s) &= \frac{eF_3(s)}{2m_\tau}, \\ d_\tau^Z(s) &= \frac{eG_3(s)}{2m_\tau}. \end{aligned} \tag{10}$$

The SM value of  $a_\tau^Z(m_Z^2)$ , which is complex, was computed in [8]. The CP-violating form factors  $d_\tau^{\gamma,Z}(s)$  are unmeasurably small within the SM, but they are generated to one-loop order in a number of SM extensions. The branch point at which these form factors develop an imaginary part depends on the model (cf. [22, 23] and below). As an example of a SM extension we mention models with neutral Higgs bosons  $\varphi$  which couple both to scalar and pseudoscalar quark and lepton currents (cf., for instance [22]). Neglecting the electron mass then to one-loop approximation  $\varphi$  exchange induces the CP-violating contributions to the S matrix element of  $e^+e^- \rightarrow \tau^+\tau^-$  depicted in Fig.2. These contributions lead to form factors  $d_\tau^\gamma(s)$  and  $d_\tau^Z(s)$ . Clearly they have imaginary parts for  $s > 4m_\tau^2$ . The contribution Fig. 2b to the WDM form factor has its branch point at  $(m_Z + m_\varphi)^2$ .

### 3 Effective Lagrangian versus Form Factors

Let us discuss now in general terms interactions beyond the SM with an intrinsic mass scale  $\Lambda$  much larger than the characteristic energy scale of a

reaction, which in our case is  $m_Z$ :

$$\Lambda \gg m_Z. \quad (11)$$

Then the effects of these new interactions at or below the  $Z$  scale can be parametrized by an effective Lagrangian which consists of a sum of hermitian local operators containing only SM fields with real coupling constants. The operators and their couplings in  $\mathcal{L}_{\text{eff}}$  can be classified according to their (mass) dimensions. After spontaneous symmetry breaking the effective Lagrangian for CP violation in the  $\gamma$  fermion and  $Z$  fermion systems reads, including operators up to dimension five [2, 24]:

$$\mathcal{L}_{\text{eff},1} = -\frac{i}{2} \bar{f} \sigma_{\mu\nu} \gamma_5 f \left[ \tilde{d}_f^{\gamma(0)} F^{\mu\nu} + \tilde{d}_f^{Z(0)} Z^{\mu\nu} \right] - \tilde{a}_f^{(0)} (\bar{f} i \gamma_5 f) Z_\mu Z^\mu, \quad (12)$$

where  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$  is the electromagnetic field strength tensor and  $Z^{\mu\nu} = \partial^\mu Z^\nu - \partial^\nu Z^\mu$ . The constants  $\tilde{d}_f^{\gamma(0)}, \tilde{d}_f^{Z(0)}, \tilde{a}_f^{(0)}$  in (12) are real. If we calculate the dipole form factors (10), which follow from (12), to zeroth order in the SM coupling constants, we get:

$$\begin{aligned} d_f^\gamma(s) &= \tilde{d}_f^{\gamma(0)}, \\ d_f^Z(s) &= \tilde{d}_f^{Z(0)}. \end{aligned} \quad (13)$$

(In fact, the electromagnetic coupling  $e$  is contained in  $\tilde{d}^{\gamma,Z(0)}$  which we do not count.) This implies in particular that the imaginary parts of the form factors (10) vanish at zeroth order. The relations (13) are modified when SM corrections are taken into account. To first and second order in the SM couplings there are 2 types of such corrections:

1) Corrections which are of first order both in the SM and in the CP-violating couplings which differ from the dipole couplings. Typical diagrams for this type of corrections are shown in Fig. 3a which involves the  $ZZ\tau\tau$  interaction of (12), and in Fig. 3b which involves the couplings (cf. [4])

$$\begin{aligned} \mathcal{L}_{\text{eff},2} = \sum_f \big\{ & \tilde{h}_{1f}^{(0)} (\bar{f} f) (\bar{\tau} i \gamma_5 \tau) \\ & + \tilde{h}_{2f}^{(0)} (\bar{f} i \gamma_5 f) (\bar{\tau} \tau) \\ & + \tilde{h}_{3f}^{(0)} \epsilon^{\mu\nu\rho\sigma} (\bar{f} \sigma_{\mu\nu} f) (\bar{\tau} \sigma_{\rho\sigma} \tau) \big\}. \end{aligned} \quad (14)$$

Here  $f$  can be any of the fundamental fermions of the SM. Actually, the scalar-pseudoscalar interactions in (14) generate EDM and WDM form factors of the  $\tau$  through diagram Fig. 3b only if  $f = \tau$ . For  $f \neq \tau$  the contribution of the first term in (14) to the diagram Fig. 3b is zero, whereas the second term in (14) leads to the scalar form factor  $G_5$  mentioned in sect. 2. For  $f = \tau$  one can use the Fierz identity  $(\bar{\tau} \tau)(\bar{\tau} \gamma_5 \tau) = -(\bar{\tau} \sigma_{\mu\nu} \tau)(\bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau)/12$ . That is, in this case only the coupling  $\tilde{h}_{3\tau}^{(0)}$  appears in (14). Fig. 3b and the analogous diagram with an external photon lead to complex WDM and EDM form factors above the  $\tau^+ \tau^-$  threshold.

2) Corrections involving the dipole couplings of (12). Typical diagrams for

this type of corrections are shown for the  $Z\tau\tau$  form factor  $d_\tau^Z(m_Z^2)$  in Fig. 4. They are of second order in the SM couplings.

It may be instructive to discuss how these diagrams come about in specific models – provided (11) applies. In models with Higgs bosons  $\varphi$  of indefinite CP parity, diagrams Figs. 3 a,b result from the vertex functions in Figs. 2 b,a, respectively, when the  $\varphi$  propagator is shrunk to a point, which is a reasonable approximation if  $m_\varphi \gg m_Z$ . In these models only the local  $ZZ\tau\tau$  of (12) and the scalar-pseudoscalar interactions of (14) are generated (to Born approximation) in this limit. Hence Figs. 3a,b, which generate the EDM and WDM form factors as discussed above, are, in fact, in these models one-loop effects, whereas Figs. 4 appear at two loops and are presumably less important.

There are also models, for instance leptoquark models, where also the local dipole interactions of (12) are generated (to one-loop approximation) in the limit (11). Then imaginary parts of the EDM and WDM form factors in the kinematic region  $s \ll \Lambda^2$  are due to Figs. 3,4, which are two-loop effects in this case and hence expected to be small.

In summary: both types of corrections 1) and 2) can lead to imaginary parts of the form factors  $d_\tau^{\gamma,Z}(m_Z^2)$ . Moreover, these corrections make it in general necessary to renormalize the coupling parameters of the effective Lagrangian (12):

$$\begin{aligned}\tilde{d}_\tau^{\gamma(0)} &\rightarrow \tilde{d}_\tau^{\gamma(\text{ren.})} \\ \tilde{d}_\tau^{Z(0)} &\rightarrow \tilde{d}_\tau^{Z(\text{ren.})}\end{aligned}$$

The parameters  $\tilde{d}_\tau^{\gamma,Z(\text{ren.})}$  are then real, finite quantities, which we can call the renormalized EDM and WDM coupling constants of the effective Lagrangian. There is a computable functional connection between the directly measurable quantities, for instance the form factors  $d_\tau^{\gamma,Z}(m_Z^2)$  and the renormalized coupling parameters of  $\mathcal{L}_{\text{eff}}$ :

$$\begin{aligned}\text{measurable quantity} &= \text{function of } (\tilde{d}_\tau^{Z(\text{ren.})}, \tilde{d}_\tau^{\gamma(\text{ren.})}, \tilde{h}_{if}^{(\text{ren.})}, \\ &\dots, \text{renormalized SM parameters}).\end{aligned}\quad (15)$$

Such relations for  $d_\tau^{\gamma,Z}(m_Z^2)$  generalize then (13) and give also imaginary parts to the form factors.

The advantage of the effective Lagrangian approach is its model-independence. It allows us to relate anomalous effects in various reactions to a small set of parameters if we restrict ourselves to operators of dimension smaller than or equal to some fixed number, e.g. six, in  $\mathcal{L}_{\text{eff}}$ . Yet using these couplings to leading order, which is often sufficient for a phenomenological analysis, the dependence of “new physics” effects on kinematic invariants is absent by construction. In the form factor approach, on the other hand, we have to introduce for each reaction a set of independent form factors which are a priori unknown functions of the relevant kinematic variables. But form factors are more directly related to the experimental observables. Fortunately, in the case at hand the EDM and WDM form factors are constant for fixed c.m.

energy. Thus a good strategy could be to have experimentalists present their results in terms of form factors and have theorists analyse the implications for the couplings of the effective Lagrangian and/or of specific models beyond the SM.

At this point we can return to the question of infrared divergences in the form factors. Due to these divergences, the form factors at  $m_Z^2$  are, strictly speaking, not directly measurable quantities. Thus, if one wants to include QED radiative corrections in a systematic way one either has to go to the effective Lagrangian approach anyway or one has to work with form factors defined at the theory level before radiative corrections. It is clear that the latter approach would be rather similar to the effective Lagrangian one. We may also note that above we treated the  $Z$  boson as an on-shell particle, which, of course, it never is. A more rigorous definition of the form factors in (8) could be given with the help of the partial wave decomposition of the amplitude for the reaction  $e^+e^- \rightarrow \tau^+\tau^-$ . But this still would leave us with the problem of infrared divergences.

Finally let us discuss the reaction  $e^+e^- \rightarrow \tau^+\tau^-$  away from the  $Z$  pole. We shall confine ourselves to a discussion of CP-violating contributions. (The question of whether or not the use of AMM and WMM form factors still provides a gauge-invariant description of CP-invariant chirality-flipping effects in the final state requires a detailed consideration within a specific model.) Rather than performing a completely model-independent analysis of this amplitude by means of a Lorentz covariant decomposition, it is more useful to analyse this process either in terms of effective Lagrangians or in terms of specific models of CP violation. We briefly sketch the latter approach. Consider extensions of the Standard Model with the same gauge boson content as the SM, and which predict, apart from the Kobayashi-Maskawa phase, also CP-violating "non universal" interactions of the Higgs boson type with couplings to fermions being proportional to the mass of a fermion involved. Such couplings induce much larger dipole form factors for heavy fermions as compared to light flavours. It is primarily the search for such interactions which makes CP studies in tau physics so interesting. Neglecting the electron mass then in these models CP violation in the above amplitude arises to one-loop approximation via induced electric and weak dipole form factors in the  $\gamma\tau^+\tau^-$  and  $Z\tau^+\tau^-$  vertex functions, respectively. A posteriori no strong-coupling CP phenomena are expected in the case at hand. Hence the one-loop approximation is sufficient. It is therefore a valid and quite general procedure to parameterize CP-violating effects in  $e^+e^- \rightarrow \tau^+\tau^-$  in terms of  $d_\tau^\gamma(s)$  and  $d_\tau^Z(s)$  as was done in [6]. It depends on the specific model whether these form factors have, for given c.m. energy  $\sqrt{s}$ , also imaginary parts. As discussed above in extensions of the SM with neutral Higgs bosons of undefined CP parity [22] the EDM form factor  $\text{Im}d_\tau^\gamma(s) \neq 0$  for  $s > 4m_\tau^2$ , whereas for instance in leptoquark models  $d_\tau^\gamma(s)$  will have a sizeable imaginary part only above the  $t\bar{t}$  threshold [23]. Depending on the c.m. energy the imaginary parts of the form factors can become larger in magnitude than the real parts. Future experiments should therefore search, as was done in [11], both for nonzero  $\text{Re}d_\tau^{\gamma,Z}(s)$  and  $\text{Im}d_\tau^{\gamma,Z}(s)$ .



## 4 An “Optimal” Strategy for Searching for CP-Violation in $Z \rightarrow b\bar{b}X$

In [15] a search for CP-violation in  $Z \rightarrow b\bar{b}X$  was suggested. This seems very worthwhile since the experimental results for the rate  $Z \rightarrow b\bar{b}X$  deviate considerably from the SM prediction (cf. e.g. [25]). The problem with which experimentalists are confronted is to select  $Z \rightarrow b\bar{b}X$  decays from all hadronic decays of the  $Z$  and to make a CP analysis. Typically one has a parameter  $\xi$  which gives information if the hadronic event is a  $Z \rightarrow b\bar{b}X$  decay. This parameter  $\xi$  could be the measured length  $L$  between the primary and the secondary vertex of the event divided by the width  $\sigma$  of the vertex distribution

$$\xi = L/\sigma. \quad (16)$$

Let  $w(\xi)$  be the probability that the event is a  $b$ -event if  $\xi$  is measured, and  $1-w(\xi)$  the probability that it is a hadronic event coming from the production of  $u, d, s, c$  quarks, denoted collectively by  $q$  in the following. The probability  $w(\xi)$  is usually well known from Monte Carlo studies. Furthermore, let  $\rho(\xi)$  be the normalized density of events on the  $\xi$ -axis

$$\int d\xi \rho(\xi) = 1. \quad (17)$$

Consider now the  $Z$ -decay distribution with respect to a set of kinematic variables  $\phi$  (typically jet variables) and  $\xi$  for all the hadronic events,  $q$  and  $b$  taken together:

$$\frac{1}{\Gamma} \frac{d\Gamma(\phi, \xi)}{d\phi d\xi} = \left\{ [1 - w(\xi)] S_{\text{SM}}^q(\phi) + w(\xi) [S_{\text{SM}}^b(\phi) + \hat{h}_b S_1^b(\phi)] \right\} \cdot \rho(\xi). \quad (18)$$

Here we assume for simplicity that only  $b$ -quarks have a CP-violating coupling to the  $Z$  which can be parametrized by a single coupling constant  $\hat{h}_b$  (cf. [15]). The quantity  $S_{\text{SM}}^q(\phi)$  describes the SM distribution in the kinematic variables  $\phi$  for  $u, d, s, c$  quarks,  $S_{\text{SM}}^b(\phi)$  for the  $b$  quarks and  $S_1^b(\phi)$  is the CP-odd interference term between the SM and CP-odd amplitudes for the  $b$  quarks. Here and in the following terms of order  $\hat{h}_b^2$  are neglected.

The problem is now to devise an “optimal” strategy to test for the presence of the  $\hat{h}_b$  term in (18). The solution to this type of problem is well known [26]. The optimal observable is

$$O = \frac{w(\xi) S_1^b(\phi)}{(1 - w(\xi)) S_{\text{SM}}^q(\phi) + w(\xi) S_{\text{SM}}^b(\phi)}. \quad (19)$$

To make things simple, let us consider the case where the SM distributions  $S_{\text{SM}}^q(\phi)$  and  $S_{\text{SM}}^b(\phi)$  are equal:

$$S_{\text{SM}}^q(\phi) = S_{\text{SM}}^b(\phi). \quad (20)$$

Usually this is a good approximation, valid to within a few percent. We get then

$$O = w(\xi) \frac{S_1^b(\phi)}{S_{\text{SM}}^b(\phi)}, \quad (21)$$

$$\begin{aligned}
\langle O \rangle &= \hat{h}_b \int d\xi \rho(\xi) w^2(\xi) \int d\phi \frac{[S_1^b(\phi)]^2}{S_{\text{SM}}^b(\phi)}, \\
\langle O^2 \rangle &= \int d\xi \rho(\xi) w^2(\xi) \int d\phi \frac{[S_1^b(\phi)]^2}{S_{\text{SM}}^b(\phi)}
\end{aligned} \tag{22}$$

and for the 1 s.d. error of an (ideal) measurement of  $\hat{h}_b$  via  $O$ :

$$(\delta \hat{h}_b)_{\text{opt}}^2 = \frac{\langle O^2 \rangle}{N(\langle O \rangle / \hat{h}_b)^2} = \left[ \int d\phi \frac{[S_1^b(\phi)]^2}{S_{\text{SM}}^b(\phi)} \right]^{-1} N_{b,\text{eff}}^{-1}. \tag{23}$$

Here

$$N_{b,\text{eff}} := N \int d\xi \rho(\xi) w^2(\xi) \tag{24}$$

and  $N$  is the total number of  $Z$  decays considered. Since we have  $0 \leq w(\xi) \leq 1$ , the total number of  $b$ -events

$$N_b = N \int d\xi \rho(\xi) w(\xi) \tag{25}$$

is always bigger or equal  $N_{b,\text{eff}}$ :

$$N_b \geq N_{b,\text{eff}}. \tag{26}$$

Thus, the error  $(\delta \hat{h}_b)_{\text{opt}}^2$  in (23) is larger or equal to the error one would have in an ideal experiment, where all  $b$ -events could be selected unambiguously, which would correspond to the replacement  $N_{b,\text{eff}} \rightarrow N_b$  in (23).

On the other hand, the procedure of making cuts on  $b$ -events in a region where  $w(\xi)$  is large, say for

$$\xi_1 \leq \xi \leq \xi_2 \tag{27}$$

and using as observable the optimal one, disregarding  $\xi$ :

$$\tilde{O} = \frac{S_1^b(\phi)}{S_{\text{SM}}^b(\phi)}, \tag{28}$$

produces an error larger or equal to the one in (23). Indeed, in this case we get for the number of  $b$ -events within the cuts (27)

$$N_{b,\text{cut}} := N \int_{\xi_1}^{\xi_2} d\xi \rho(\xi) w(\xi) \tag{29}$$

and for the expectation values of  $\tilde{O}$ ,  $\tilde{O}^2$  and for  $(\delta \hat{h}_b)_{\text{cut}}^2$ :

$$\langle \tilde{O} \rangle_{\text{cut}} = \hat{h}_b \frac{N_{b,\text{cut}}}{N} \int d\phi \frac{[S_1^b(\phi)]^2}{S_{\text{SM}}^b(\phi)}, \tag{30}$$

$$\langle \tilde{O}^2 \rangle_{\text{cut}} = \int_{\xi_1}^{\xi_2} d\xi \rho(\xi) \int d\phi \frac{[S_1^b(\phi)]^2}{S_{\text{SM}}^b(\phi)}, \tag{31}$$

$$(\delta \hat{h}_b)_{\text{cut}}^2 = \left[ \int d\phi \frac{[S_1^b(\phi)]^2}{S_{\text{SM}}^b(\phi)} \right]^{-1} \tilde{N}_{b,\text{cut}}^{-1}, \tag{32}$$

where

$$\tilde{N}_{b,cut} := N_{b,cut}^2 \left[ N \cdot \int_{\xi_1}^{\xi_2} d\xi \rho(\xi) \right]^{-1}. \quad (33)$$

Using the Cauchy-Schwartz inequality we find easily

$$\begin{aligned} N_{b,cut}^2 &\leq N^2 \int_{\xi_1}^{\xi_2} d\xi' \rho(\xi') \int_{\xi_1}^{\xi_2} d\xi \rho(\xi) w^2(\xi), \\ N_{b,cut}^2 \left[ N \int_{\xi_1}^{\xi_2} d\xi \rho(\xi) \right]^{-1} &\leq N \int_{\xi_1}^{\xi_2} d\xi \rho(\xi) w^2(\xi) \\ &\leq N \int d\xi \rho(\xi) w^2(\xi). \end{aligned} \quad (34)$$

Thus

$$\tilde{N}_{b,cut} \leq N_{b,eff}, \quad (35)$$

$$(\delta \hat{h}_b)_{cut}^2 \geq (\delta \hat{h}_b)_{opt}^2, \quad (36)$$

q.e.d.

## 5 Conclusions

We hope that this note will be useful for experimentalists and interesting for theorists.

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## Figure Captions

- Fig. 1 Photon exchange diagram which contains the  $ff\gamma$ -vertex (1).
- Fig. 2 CP-violating contributions to  $e^+e^- \rightarrow \tau^+\tau^-$  (for  $m_e=0$ ) in models with Higgs bosons  $\varphi$  of undefined CP parity. Diagrams with permuted vertices are not drawn.
- Fig. 3 Diagrams which involve the CP-odd  $ZZ\tau\tau$  coupling (a) and the CP-odd 4-point couplings (b) which give a contribution to the dipole form factor  $d_\tau^Z(m_Z^2)$ .
- Fig. 4 Radiative corrections to the  $Z\tau\tau$  dipole coupling in the SM.

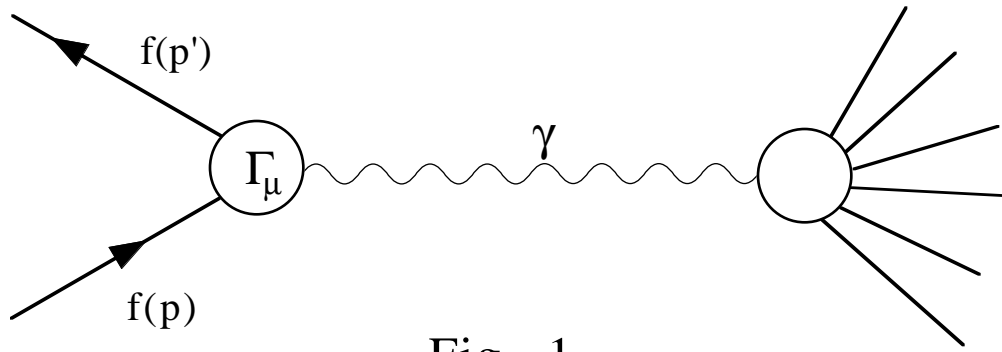
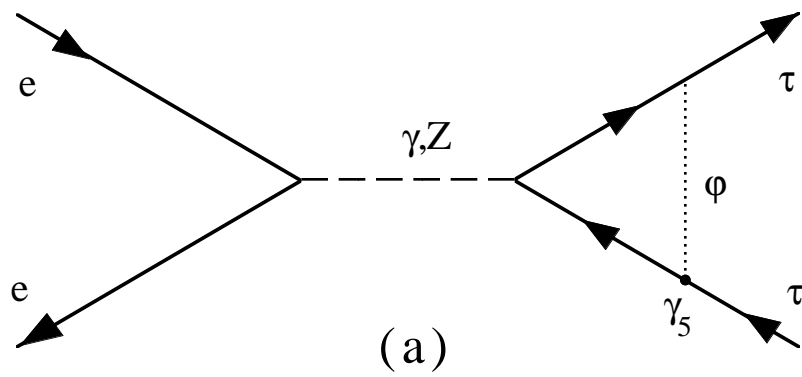
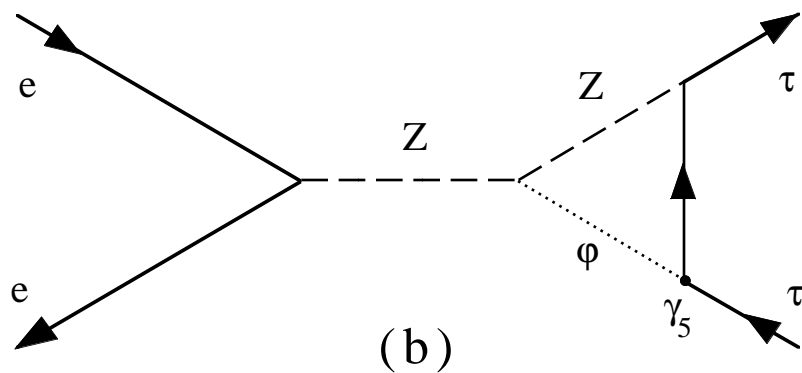


Fig. 1

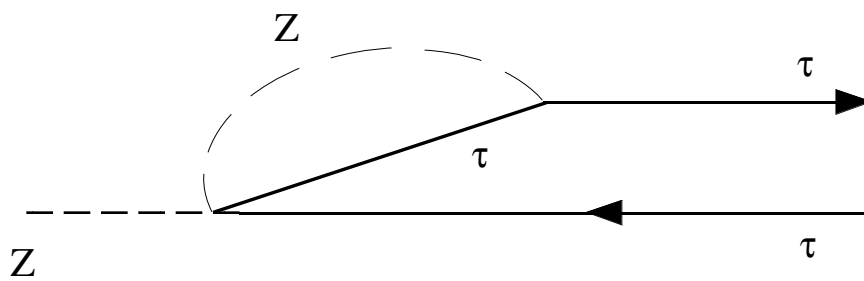


(a)

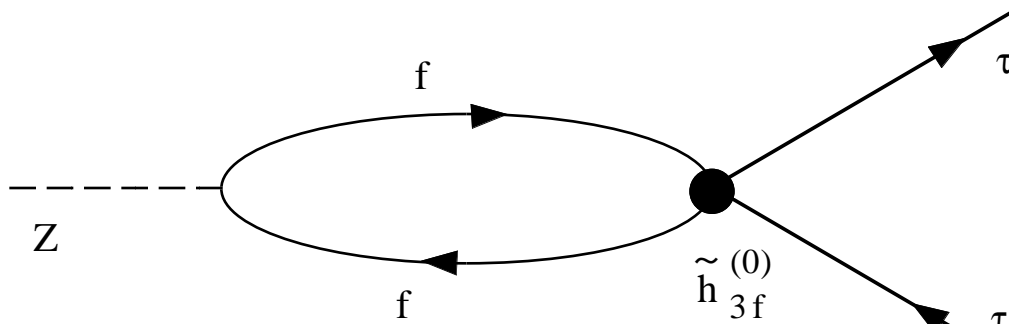


(b)

Fig. 2



( a )



( b )

Fig. 3

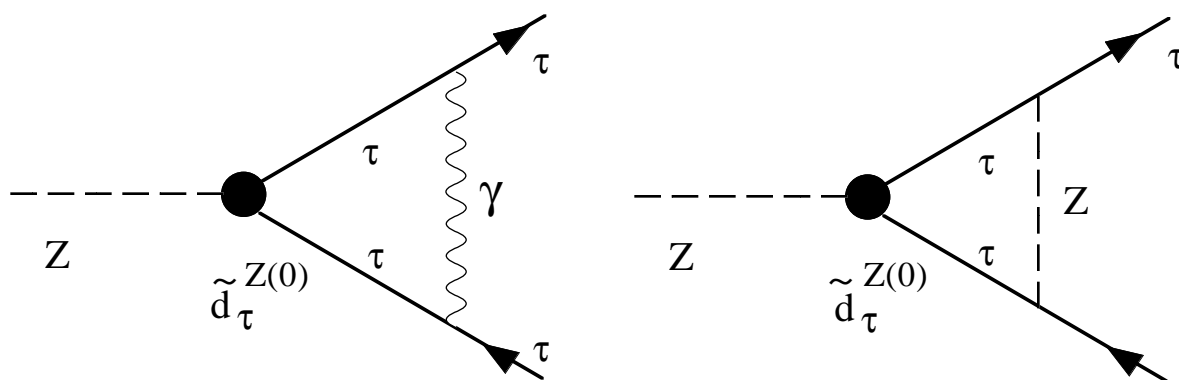


Fig. 4